

1. 900

2. Suppose our digits are A, B, C then our number can be represented as $N = 100A + 10B + C = 4ABC$. We need to use our deduction skills to examine this information to narrow down our possibilities and then do some guessing and checking.

(1) First of all, notice that this means that our number N has to be divisible by 4. In particular this helps us because this tells us $10B + C$ has to be divisible by 4.

Since our first bit of information tells us about the last two digits, let A depend on B, C by solving for A :

$$\begin{aligned} 100A + 10B + C &= 4ABC \\ 10B + C &= 4ABC - 100A \\ 10B + C &= A(4BC - 100) \\ \frac{10B + C}{4BC - 100} &= A \\ \frac{10B + C}{4(BC - 25)} &= A \end{aligned}$$

(2) To further narrow down B, C , we can say that $BC - 25$ divides $10B + C$.

(3) The last bit of information that we can deduce is that since N is a three-digit number, $100 \leq 4ABC < 1000$ or $25 \leq BC < 250$ (since A is some positive integer).

Now let's begin looking at our number $10B + C$, which must be a multiple of 4. I will only go through the first few multiples of 4 and let you check the rest up to the answer.

04, 08, 12, ..., 44: $BC < 25$ and does not meet requirement 3

48: $BC - 25 = 32 - 25 = 7$ does not divide $10B + C = 48$ and does not meet requirement 2

52, 56, 60, 64: $BC < 25$ and does not meet requirement 3

68: $BC - 25 = 48 - 25 = 23$ does not divide $10B + C = 68$ and does not meet requirement 2

72: $BC < 25$ and does not meet requirement 3

76: $BC - 25 = 42 - 25 = 17$ does not divide $10B + C = 76$ and does not meet requirement 2

80: $BC < 25$ and does not meet requirement 3

84: $BC - 25 = 32 - 25 = 7$ divides $10B + C = 84$ and $25 \leq BC = 32 < 250$, meeting both requirements.

Our equation gives us $A = 3$ and a resulting number, $N = 384$.

3. In order for $\sqrt{18 \cdot n \cdot 34}$ to be an integer, $18 \cdot n \cdot 34$ must be a perfect square. We will find the smallest positive integer value of n to make a perfect square. First we want to find the prime factorization of $18 \cdot 34 = 2^2 \cdot 3^2 \cdot 17$. This means that $\sqrt{18 \cdot n \cdot 34} = 6\sqrt{17n}$ so the smallest n we're looking for is 17.

4. 5



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5. There are three possible triangles, with three possible perimeters:

(1) $3x + 62 = 7x + 30$

$x = 8$ so our sides have length $3(8) + 62 = 86$, 86 , $5(8) + 50 = 90$. The perimeter of this triangle is $2(86) + 90 = 262$.

(2) $3x + 62 = 5x + 50$

$x = 6$ so our sides have length $3(6) + 62 = 80$, 80 , $7(6) + 30 = 72$. The perimeter of this triangle is $2(80) + 72 = 232$.

(3) $7x + 30 = 5x + 50$

$x = 10$ so our sides have length $3(10) + 62 = 92$, $7(10) + 30 = 100$, 100 . The perimeter of this triangle is $2(100) + 92 = 292$.

The least possible perimeter is $\boxed{232}$ of triangle (2).

6. $\boxed{36}$

7. $\frac{1}{3} + \frac{1}{5} + \frac{1}{6} + \frac{1}{4} = \frac{57}{60}$ of the coins were given away. This means Troy's six coins are $1 - \frac{57}{60} = \frac{1}{20}$ of the coins. This means that there must be a total of $6 \cdot 20 = \boxed{120}$ coins.

8. $\boxed{27}$

9. $\boxed{19}$

10. Together they have $87 + 213 = 300$ dimes and $213 + 87 = 300$ quarters. This amounts to $300 \cdot \$0.10 + 300 \cdot \$0.25 = \boxed{\$105}$